Electric permittivity $E0_meters := 8.854 \cdot 10^{-12}$ F/m of free space (metric)

Recalculate in in.

E0_inches := E0_meters
$$.0254$$

E0_inches = 2.249×10^{-13}

U0 inches := U0 meters .0254

U0_inches = 3.192×10^{-8}

Display calculated value

Magnetic permeability $U0_meters := 4 \cdot \pi \cdot 10^{-7}$ H/m of free space (metric)

Recalculate in in.

Display calculated value

We often need this number $\frac{U0_inches}{2 \cdot \pi} = 5.08 \times 10^{-9}$

Speed of light C_meters := 2.998 · 10⁸ m/s (metric)

Recalculated in in.

 $C_inches = 1.18 \times 10^{10}$

C_inches := $\frac{C_meters}{.0254}$

Display calculated value

$$\frac{10^{12}}{\text{C_inches}} = 84.723$$

Propagation delay at light speed (ps/in.)

CONSTANT.MCD -- 9/7/2001 -- p.1

file: resist.mcd DC RESISTANCE OF COPPER WIRES AND TRACES Conversion formulas included in this spreadsheet: Diameter to AWG AWG() AWG to diameter DIAMETER() Thickness to copper plating weight CPW() Copper plating weight to thickness THICKNESS() Resistance formulas included in this spreadsheet: DC resistance of round wires From diameter RROUND() From AWG wire size RROUND AWG() At room temperature only RROUND RT() DC resistance of printed circuit board traces From trace thickness and width RTRACE() RTRACE CPW() Using copper plating weight At room temperature only RTRACE RT() DC resistance of power or ground planes

Using thickness and via diameter RPLANE() Using copper plating weight RPLANE CPW()

Variables used:

 ρ Bulk resistivity of copper

 $\rho := 6.787 \cdot 10^{-7}$ ohm-in.

This coefficient is slightly different from the bulk resistivity of pure copper (6.58E-07) owing to the annealing process used in making wire, and chemical imperfections in the copper used for making practical wires.

In practice, the resistance of two wires making up a twisted pair may often be matched as well as 10%, but almost never as well as 1%.

 $\delta \rho$ Thermal coefficient of resistance $\delta \rho := .0039$ per deg. C If the resistance of a copper wire is R at room temperature, then at a temperature $1\,^{\circ}\text{C}$ higher it will be R(1 + $\delta\rho)$. This coefficient applies to standard annealed copper wires. The coefficient for pure copper in its bulk state varies slightly. Over a temperature range 0-70°C the resistance of copper wires varies 28%. Length of wire (in.) х (or separation between contact points on ground plane) Diameter of wire (in.) d (or diameter of contact point on ground plane) AWG American wire gauge (English units) temp Temperature (°C) Width of printed circuit board trace (in.) W Thickness of printed circuit board trace (in.) t cpw Thickness of printed circuit board traces, in units of copper plating weight (oz/ft^2)

Conversions between American $-\frac{awg+10}{20}$ Wire Gauge (AWG) and diameter (in.): DIAMETER(awg) := 10

 $AWG(d) := -10 - 20 \cdot \log(d)$

General formula for resistance of a round wire (Ω) :

RROUND(d, x, temp) :=
$$\frac{4 \cdot \rho \cdot x}{\pi \cdot d^2} \cdot \left[1 + (\text{temp} - 20) \cdot \delta \rho\right]$$

RESIST.MCD -- 9/7/2001 -- p.2

Resistance of a round wire specified by AWG size instead of diameter (Ω):

RROUND_AWG(awg, x, temp) := RROUND(DIAMETER(awg), x, temp)

Resistance of a round wire at room temperature (Ω) :

 $RROUND_RT(d, x) := RROUND(d, x, 20)$

Conversion between thickness, t (in.) and copper plating weight, cpw (oz):

 $CPW(t) := \frac{t}{.00137}$

THICKNESS(cpw) := .00137.cpw

Resistance of a circuit trace (Ω) :

 $RTRACE(w, t, x, temp) := \frac{x \cdot \rho}{w \cdot t} \cdot \left[1 + (temp - 20) \cdot \delta \rho\right]$

Resistance of a trace specified by plating weight instead of thickness (Ω):

RTRACE_CPW(w, cpw, x, temp) := RTRACE(w, THICKNESS(cpw), x, temp)

Resistance of a circuit trace at room temperature $\left(\Omega\right)$:

 $RTRACE_RT(w, t, x) := RTRACE(w, t, x, 20)$

RESIST.MCD -- 9/7/2001 -- p.3

Resistance of a power or ground plane (Ω) :

When using long, skinny traces or wires, the approximations above work extremely well. Each formula assumes a uniform distribution of current throughout the conducting body, for which resistance is directly proportional to length.

Currents circulating in a large ground or power plane are not uniform. Consequently, the resistance measured between two points on a ground or power plane is not directly proportional to the separation between measurement points.

The following equation models the resistance between two contact points on a ground plane. This model assumes each contact point touches the ground plane over some finite area. The approximate diameter of the contact point determines the overall resistance.

If the contact points lie near any edge of the plane, the resistance between them may go up by a factor of 2. The resistance near corners may rise even higher.

d1	Diameter of 1st contact point (in.)
d2	Diameter of 2nd contact point (in.)
t	Thickness of plane (in.)
срw	Thickness of plane, copper plating weight (oz)
x	Separation between contact points (in.)
temp	Temperature (°C)

Resistance of a power or ground plane (Ω) :

 $RPLANE(d1, d2, t, x, temp) := \frac{\rho}{2 \cdot \pi \cdot t} \cdot \left(ln \left(\frac{2 \cdot x}{d1} \right) + ln \left(\frac{2 \cdot x}{d2} \right) \right) \cdot \left[1 + (temp - 20) \cdot \delta \rho \right]$

Resistance of a power or ground plane specified by plating weight instead of thickness (Ω):

RPLANE_CPW(d1,d2,cpw,x,temp) := RPLANE(d1,d2,THICKNESS(cpw),x,temp)

CAPACITANCE OF TWO PARALLEL PLATES

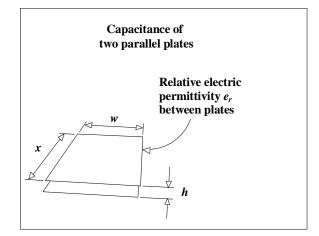
Formulas included in this spreadsheet:

Capacitance of two plates	CPLATE()
Impedance magnitude of capacitor at	
one frequency	XCF()
Impedance magnitude of capacitor as	
seen by rising edge	XCR()

w Width of plate overlap
(in.)

x Length of plate overlap
(in.)

- h Height of one plate above the other (in.)
- er Relative dielectric constant of material between plates



capac

Capacitance of two plates (F):

$$CPLATE(w, x, h, er) \coloneqq 2.249 \cdot 10^{-13} \cdot \frac{er \cdot x \cdot w}{h}$$

A power and ground plane separated by 0.010 in. of FR-4 dielectric (er = 4.5) share a capacitance of 100 pF/in.²

Halving the separation doubles the capacitance.

Impedance magnitude of capacitor at frequency f $\left(\,\Omega\right)$:

- c Capacitance (F)
- f Frequency (Hz)

$$XCF(c, f) := \frac{1}{2 \cdot \pi \cdot f \cdot c}$$

The impedance, at 100 MHz, of a 100-pF capacitor is 16 $\Omega.$ $\mathrm{XCF}\!\!\left(100\!\cdot\!10^{-12},10^8\right) = 15.915$

Impedance magnitude of capacitor as seen by rising edge $\$ ($\Omega)$:

c Capacitance (F)

tr 10-90% rise time (s)

$$XCR(c,tr) := \frac{tr}{\pi \cdot c}$$

The impedance, as seen by a 5-ns rising edge of a 100-pF capacitor is 16 $\Omega.$

 $\operatorname{XCR}\left(100 \cdot 10^{-12}, 5 \cdot 10^{-9}\right) = 15.915$

INDUCTANCE OF CIRCULAR LOOP

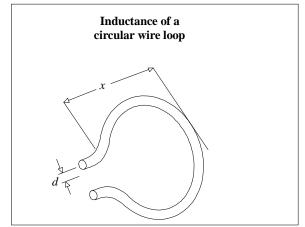
```
file: circular.mcd
```

Formulas included in this spreadsheet:

Inductance of circular wire loop	LCIRC()
Impedance magnitude of inductor at	
one frequency	XLF()
Impedance magnitude of inductor as	
seen by rising edge	XLR()

Variables used:

- d Diameter of wire (in.)
- x Diameter of wire loop (in.)



circular

Inductance of wire loop (H):

$$\text{LCIRC}(d, x) \coloneqq 1.56 \cdot 10^{-8} \cdot x \cdot \left(\ln \left(\frac{8 \cdot x}{d} \right) - 2 \right)$$

A loop of 24-gauge wire the size of the loop between your thumb and forefinger has about 100 nH of inductance. $LCIRC(.01, 1.3) = 1.003 \times 10^{-7}$

Changing the wire diameter from AWG 24 to AWG 14 makes little difference. The log function is rather insensitive to wire size. $LCIRC(.1,1.3) = 5.363 \times 10^{-8}$ Impedance magnitude of inductor at frequency f $\left(\,\Omega\right)$:

- 1 Inductance (H)
- f Frequency (Hz)

$$XLF(1, f) := 2 \cdot \pi \cdot f \cdot l$$

The impedance, at 100 MHz, of a 100-nH inductor is 62 $\Omega.$

$$\mathrm{XLF}\left(100 \cdot 10^{-9}, 10^{8}\right) = 62.832$$

Impedance magnitude of inductor as seen by rising edge ($\Omega)$:

1 Inductance (H)

tr 10-90% rise time (s)

$$XLT(1, tr) := \frac{\pi \cdot l}{tr}$$

The impedance, as seen by a 5-ns rising edge, of a 100-nH inductor is 62 $\Omega\,.$

$$XLT(100 \cdot 10^{-9}, 5 \cdot 10^{-9}) = 62.832$$

INDUCTANCE OF RECTANGULAR LOOPS

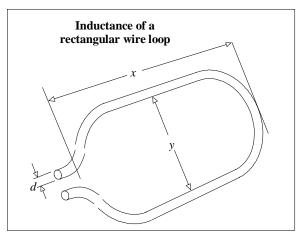
```
file: rectangl.mcd
```

Formulas included in this spreadsheet:

Inductance of rectangular wire loop
Impedance magnitude of inductor at
 one frequency
Impedance magnitude of inductor to
 rising edge

Variables used:

- d Diameter of wire (in.)
- x Length of wire loop (in.)
- y Breadth of wire loop (in.)



LRECT()

XLF()

XLR()

rectangle

Inductance of wire loop (H):

LRECT(d, x, y) :=
$$10.16 \cdot 10^{-9} \cdot \left(x \cdot \ln\left(\frac{2 \cdot y}{d}\right) + y \cdot \ln\left(\frac{2 \cdot x}{d}\right) \right)$$

A loop of 24-gauge wire 1 in.² has about 100 nH of inductance.

Changing the wire diameter from AWG 30 to AWG 10 makes little difference. The log function is very insensitive to wire size.

If your loop consists of different-sized conductors, use the diameter of the smallest one.

Impedance magnitude of inductor at frequency f $\left(\,\Omega\right)$:

1 Inductance (H)

 $XLF(1, f) := 2 \cdot \pi \cdot f \cdot l$

f Frequency (Hz)

The impedance, at 100 MHz, of a 100-nH inductor is 62 $\Omega.$

RECTANGL.MCD -- 9/7/2001 -- p.1

Impedance magnitude of inductor as seen by rising edge ($\Omega)$:

1 Inductance (H)

tr 10-90% rise time (s)

$$XLT(1, tr) := \frac{\pi \cdot 1}{tr}$$

The impedance, as seen by a 5-ns rising edge, of a 100-nH inductor is 62 $\Omega.$

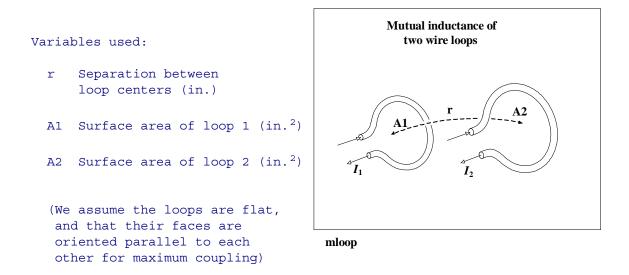
MUTUAL INDUCTANCE OF TWO LOOPS

file: mloop.mcd

Formulas included in this spreadsheet:

Mutual inductance of two loops

MLOOP()



The loops must be well separated for the MLOOP() approximation to work:

$$r > \sqrt{A1}$$
 $r > \sqrt{A2}$ and

Mutual inductance of two well-separated loops $(\ensuremath{\texttt{nH}}):$

$$MLOOP(r, A1, A2) \coloneqq 5.08 \cdot \frac{A1 \cdot A2}{r^3}$$

MLOOP.MCD -- 9/7/2001 -- p.1

MUTUAL INDUCTANCE OF PARALLEL TRANSMISSION LINES

Formulas included in this spreadsheet:

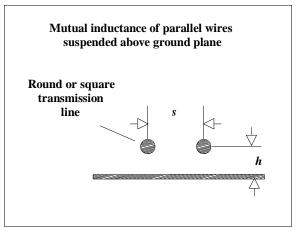
Mutual inductance of two lines

MLINE()

Variables used:

- s Separation between
 wire centers (in.)
- h Height of wires above ground (in.)
- x Length of parallel span (in.)

(We assume that two identical transmission lines share a parallel run of length x, with a horizontal separation s.)



mline

Let L equal the inductance (H) of the first transmission line of length x (use formula for round, microstrip, or stripline geometry as appropriate):

$$MLINE(L, s, h) := L \cdot \left[\frac{1}{1 + \left(\frac{s}{h}\right)^2}\right]$$

MLINE.MCD -- 9/7/2001 -- p.1

COAXIAL TRANSMISSION LINE

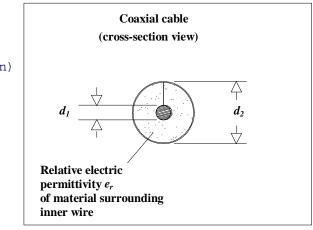
file: coax.mcd

Formulas included in this spreadsheet:

nce ZCOAX()
PCOAX ()
LCOAX()
CCOAX ()

Variables used:

- d1 Diameter of inner wire (in)
- d2 Diameter of outer shield (in)
 x Length of cable (in)
- er Relative dielectric constant of material surrounding the inner wire



coax

Characteristic impedance of coaxial cable (Ω):

$$ZCOAX(d1, d2, er) := \frac{60}{\sqrt{er}} \cdot ln\left(\frac{d2}{d1}\right)$$

Propagation delay per in. for coaxial cable (s/in.):

$$PCOAX(er) := 84.72 \cdot 10^{-12} \cdot \sqrt{er}$$

Inductance of coaxial cable (H):

$$LCOAX(d1, d2, x) := x \cdot 5.08 \cdot 10^{-9} \cdot \ln\left(\frac{d2}{d1}\right)$$

Capacitance of coaxial cable (F):

$$CCOAX(d1, d2, er, x) := \left(\frac{x \cdot 1.41 \cdot 10^{-12}}{\ln\left(\frac{d2}{d1}\right)}\right) \cdot er$$

COAX.MCD -- 9/7/2001 -- p.1

Diameter of AWG 30 inner wire (in.) D1 := .01Inside diameter of shield (in.) D2 := .1Length of cable (in.) X := 20.000Relative dielectric constant er := 2.2

Characteristic impedance (Ω):

ZCOAX(D1, D2, er) = 93.144

Total inductance (H):

$$LCOAX(D1, D2, X) = 2.339 \times 10^{-7}$$

Same result in nH:

 $LCOAX(D1, D2, X) \cdot 10^9 = 233.943$

Inductance per in. (H):

$$LCOAX(D1, D2, 1) = 1.17 \times 10^{-8}$$

Total capacitance (F):

 $CCOAX(D1, D2, er, X) = 2.694 \times 10^{-11}$

Same result in pF:

$$CCOAX(D1, D2, er, X) \cdot 10^{12} = 26.944$$

Capacitance per in. (F):

 $CCOAX(D1, D2, er, 1) = 1.347 \times 10^{-12}$

COAX.MCD -- 9/7/2001 -- p.2

TRANSMISSION LINE MADE FROM ROUND WIRE (WIRE-WRAP)

```
file: round.mcd
```

Formulas included in this spreadsheet:

Round wire characteristic impedance	ZROUND()
Round wire propagation delay	PROUND()
Round wire inductance	LROUND()
Round wire capacitance	CROUND()

Variables used:

- d Diameter of wire (in.)
- h Height of wire above ground (in.)
- x Length of wire (in.)
- (We assume the wire is suspended in air, for which the relative dielectric constant is 1.00.)

Characteristic impedance of round wire above ground plane (Ω) :

 $ZROUND(d,h) := 60 \cdot \ln\left(\frac{4 \cdot h}{d}\right)$

Propagation delay per in. of round wire above ground plane (s/in):

$$PROUND(d,h) := 84.72 \cdot 10^{-12}$$

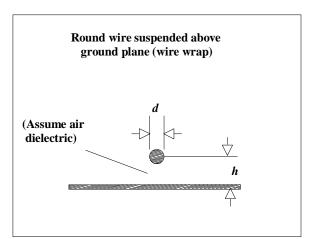
Inductance of round wire
above ground plane (H):

LROUND(d,h,x) :=
$$x \cdot 5.08 \cdot 10^{-9} \cdot \ln\left(\frac{4 \cdot h}{d}\right)$$

Capacitance of round wire above ground plane (F):

$$CROUND(d,h,x) := \left(\frac{x \cdot 1.413 \cdot 10^{-12}}{\ln\left(\frac{4 \cdot h}{d}\right)}\right)$$

ROUND.xmcd -- 1/23/2011 -- p.1



round

Example round wire calculations

Diameter of AWG 30 wire (in.) D := .01Length of wire (in.) X := 2.000Height above ground (in.) H := .100

Characteristic impedance (Ω) :

ZROUND(D, H) = 221.333

Total inductance (H):

LROUND(D, H, X) =
$$3.748 \times 10^{-8}$$

Same result in nH:

 $LROUND(D, H, X) \cdot 10^9 = 37.479$

Inductance per in. (H):

LROUND(D, H, 1) =
$$1.874 \times 10^{-8}$$

Total capacitance (F):

$$CROUND(D, H, X) = 7.661 \times 10^{-13}$$

Same result in units pF:

 $CROUND(D, H, X) \cdot 10^{12} = 0.766$

Capacitance per in. (F):

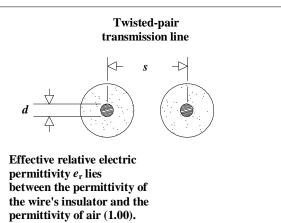
 $CROUND(D,H,1) = 3.83 \times 10^{-13}$

Formulas included in this spreadsheet:

Twisted-pair	characteristic impedance	ZTWIST()
Twisted-pair	propagation delay	PTWIST()
Twisted-pair	inductance	LTWIST()
Twisted-pair	capacitance	CTWIST()

Variables used:

- d Diameter of wire (in.)
- s Separation between
 wires (in.)
- x Length of wire (in.)
- er Effective relative dielectric constant of medium between wires



twist

Characteristic impedance of twisted pair (Ω) :

$$ZTWIST(d, s, er) := \frac{120}{\sqrt{er}} \cdot ln \left(\frac{2 \cdot s}{d}\right)$$

Propagation delay per in. twisted pair (s/in.):

$$PTWIST(er) := 84.72 \cdot 10^{-12} \cdot \sqrt{er}$$

Inductance of twisted pair (H):

Capacitance of twisted pair (F):

CTWIST(d, s, er, x) :=
$$\left(\frac{x \cdot .7065 \cdot 10^{-12}}{\ln\left(\frac{2 \cdot s}{d}\right)}\right) \cdot \text{er}$$

TWIST.MCD -- 9/7/2001 -- p.1

Example twisted-pair calculations

Diameter of AWG 24 wire (in.) D := .02Length of wire (in.) X := 2.000Separation between wire S := .038centers (in.)

er := 2.5

Relative dielectric constant

Characteristic impedance (Ω) :

$$ZTWIST(D, S, er) = 101.319$$

Total inductance (H):

LTWIST(D, S, X) = 2.713×10^{-8}

Same result in nH:

 $LTWIST(D, S, X) \cdot 10^9 = 27.127$

Inductance per in. (H):

LTWIST(D, S, 1) =
$$1.356 \times 10^{-8}$$

Total capacitance (F):

CTWIST(D, S, er, X) =
$$2.646 \times 10^{-12}$$

Same result in pF:

$$CTWIST(D, S, er, X) \cdot 10^{12} = 2.646$$

Capacitance per in. (F):

$$CTWIST(D, S, er, 1) = 1.323 \times 10^{-12}$$

TWIST.MCD -- 9/7/2001 -- p.2

MICROSTRIP TRANSMISSION LINES

```
file: mstrip.mcd
```

Formulas included in this spreadsheet:

Effective relative permittivity	EEFF()	(used internally)
Effective electrical trace width	WE()	(used internally)

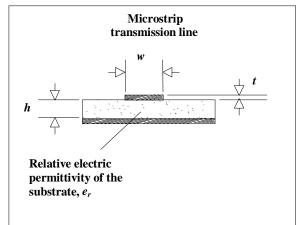
Microstrip characteristic impedance Microstrip propagation delay Microstrip trace inductance Microstrip trace capacitance

ZMSTRIP()
PMSTRIP()
LMSTRIP()
CMSTRIP()

Formulas from: I. J. Bahl and Ramesh Garg, "Simple and accurate formulas for microstrip with finite strip thickness", Proc. IEEE, 65, 1977, pp. 1611-1612.

This material is nicely summarized in T. C. Edwards, "Foundations of Microstrip Circuit Design," John Wiley, New York, 1981, reprinted 1987.

(Watch out for Edward's error in Equation 3.52b, where he omits a ln() function.)



mstrip

Variables used:

- h Trace height above ground (in.)
- w Trace width (in.)
- t Trace thickness (in.)
- er Relative permittivity of material between trace and ground plane (dimensionless)
- x Trace length (in.)

Effective relative permittivity as a function of microstrip trace geometry:

For skinny traces (w < h)

$$E_skny(h, w, er) := \frac{er + 1}{2} + \left(\frac{er - 1}{2}\right) \cdot \left[\left(1 + \frac{12 \cdot h}{w}\right)^{-1.500} + .04 \cdot \left(1 - \frac{w}{h}\right)^{2} \right]$$

For wide traces
$$(w > h)$$

E_wide(h,w,er) := $\frac{er + 1}{2} + \left(\frac{er - 1}{2}\right) \cdot \left(1 + \frac{12 \cdot h}{w}\right)^{-.500}$

Composite formula picks skinny or wide model depending on w/h ratio:

 $E_{temp}(h, w, er) := if(w > h, E_wide(h, w, er), E_skny(h, w, er))$

Special adjustment to account for trace thickness:

$$\text{EEFF}(h, w, t, er) \coloneqq \text{E_temp}(h, w, er) - \frac{(er - 1) \cdot \left(\frac{t}{h}\right)}{4.6 \cdot \sqrt{\frac{w}{h}}}$$

When w/h is skinny, you get the average of the PCB permittivity, er, and the permittivity of air. When w/h is wide, (the trace is very close to the ground plane) you get er.

Effective trace width as a function of other parameters (in.):

For skinny traces $(2\pi w < h)$

WE_skny(h, w, t) := w +
$$\frac{1.25 \cdot t}{\pi} \cdot \left(1 + \ln\left(\frac{4 \cdot \pi \cdot w}{t}\right)\right)$$

For wide traces $(2\pi w > h)$

WE_wide(h, w, t) := w +
$$\frac{1.25 \cdot t}{\pi} \cdot \left(1 + \ln\left(\frac{2 \cdot h}{t}\right)\right)$$

Composite formula picks skinny or wide model depending on w/h ratio:

WE(h, w, t) := if
$$\left(w > \frac{h}{2 \cdot \pi}, WE_wide(h, w, t), WE_skny(h, w, t) \right)$$

Characteristic impedance as a function of trace geometry (Ω) :

Accuracy of better than 2 percent is obtained under the following conditions:

 $\begin{array}{rrrr} 0 & < t/h < 0.2 \\ 0.1 & < w/h < 20 \\ 0 & < er < 16 \end{array}$

For skinny traces (w < h)

$$ZMS_skny(h, w, t) := 60 \cdot ln \left(\frac{8 \cdot h}{WE(h, w, t)} + \frac{WE(h, w, t)}{4 \cdot h} \right)$$

For wide traces (w > h)

ZMS_wide(h, w, t) := $\frac{120 \cdot \pi}{\frac{WE(h, w, t)}{h} + 1.393 + .667 \cdot \ln\left(\frac{WE(h, w, t)}{h} + 1.444\right)}$

Composite formula picks skinny or wide model depending on w/h ratio:

 $ZMSTRIP(h, w, t, er) := \frac{if(w > h, ZMS_wide(h, w, t), ZMS_skny(h, w, t))}{\sqrt{EEFF(h, w, t, er)}}$

MSTRIP.MCD -- 9/7/2001 -- p.4

Microstrip propagation delay (s/in.):

$$PMSTRIP(h, w, t, er) := 84.72 \cdot 10^{-12} \cdot \sqrt{EEFF(h, w, t, er)}$$

Inductance of microstrip (H):

 $LMSTRIP(h, w, t, x) := PMSTRIP(h, w, t, 1.) \cdot ZMSTRIP(h, w, t, 1.) \cdot x$

(Use a dummy er value of 1. It doesn't matter for inductance calculations.)

Capacitance of microstrip (F):

 $CMSTRIP(h, w, t, er, x) \coloneqq \frac{PMSTRIP(h, w, t, er)}{ZMSTRIP(h, w, t, er)} \cdot x$

MSTRIP.MCD -- 9/7/2001 -- p.5

Example microstrip wire calculations

Height above ground (in.)	H := .006	
Width of trace (in.)	W := .008	
Thickness of trace (in.)	T := .00137	(1-oz copper plating weight)
Length of wire (in.)	X := 11.000	pracing wergine,

Relative electric permeability (affects capacitance, but not inductance) er := 4.5

Impedance (Ω) :

ZMSTRIP(H, W, T, er) = 56.4435

Total inductance (H):

LMSTRIP(H, W, T, X) = 9.3401×10^{-8}

Same result in nH:

LMSTRIP(H, W, T, X)
$$\cdot 10^9 = 93.4008$$

Inductance per in. (H):

LMSTRIP(H, W, T, 1) =
$$8.491 \times 10^{-9}$$

Total capacitance (F):

CMSTRIP(H, W, T, er, X) =
$$2.9317 \times 10^{-11}$$

Same result in pF:

CMSTRIP(H, W, T, er, X)
$$\cdot 10^{12} = 29.3172$$

Capacitance per in. (F):

CMSTRIP(H, W, T, er, 1) =
$$2.6652 \times 10^{-12}$$

MSTRIP.MCD -- 9/7/2001 -- p.6

Tolerance effects

$$ZMSTRIP_TOL(h,dh,w,dw,t,er,der) := \begin{pmatrix} ZMSTRIP(h + dh,w - dw,t,er - der) \\ ZMSTRIP(h,w,t,er) \\ ZMSTRIP(h - dh,w + dw,t,er + der) \end{pmatrix}$$

 $\alpha := \text{ZMSTRIP}_\text{TOL}(.007, .002, .011, .002, .0022, 4.5, .1)$

$$\alpha = \begin{pmatrix} 64.7868\\51.3724\\37.9267 \end{pmatrix}$$

$$\operatorname{REFL}(x,z) := \begin{pmatrix} \frac{z - x_0}{z + x_0} \\ \frac{z - x_1}{z + x_1} \\ \frac{z - x_2}{z + x_2} \end{pmatrix}$$

$$\text{REFL}(\alpha, 50) = \begin{pmatrix} -0.1288 \\ -0.0135 \\ 0.1373 \end{pmatrix}$$

STRIPLINE TRANSMISSION LINES

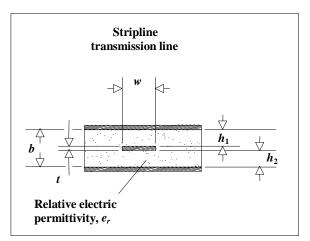
```
file: sline.mcd
```

Formulas included in this spreadsheet:

Stripline characteristic impedance	ZSTRIP()
Offset stripline characteristic impedance	ZOFFSET()
Stripline propagation delay	PSTRIP()
Stripline trace inductance	LSTRIP()
Offset stripline inductance	LOSTRIP()
Stripline trace capacitance	CSTRIP()
Offset stripline capacitance	COSTRIP()

Formulas are from Seymour Cohn, "Problems in Strip Transmission Lines," MTT-3, No. 2, March 1955, pp. 199-126.

This material is summarized in Harlan Howe, Stripline Circuit Design, Artech House, Norwood, MA, 1974.



sline

Variables used:

- h1 Trace height above lower ground plane (in.)
- h2 Trace headroom below upper ground plane (in.)
- b Separation between ground planes, b = h1 + h2 + t (in.)
- w Trace width (in.)
- t Trace thickness (in.)
- er Trace thickness (in.)
- x Trace length (in.)

Stripline characteristic impedance (Ω :)

Accuracy of better than 1.3% is obtained under the following conditions:

t/b < 0.25	NOTE: formula ZSTR_K1()
t/w < 0.11	corrected per instructions
er unrestricted	from Robert Canright of
	Richardson, TX. Thanks, Robert.

For skinny traces
$$(w/b < 0.35)$$

ZSTR_K1 $(w,t) := \left(\frac{w}{2}\right) \cdot \left[1 + \frac{t}{\pi \cdot w} \cdot \left(1 + \ln\left(\frac{4 \cdot \pi \cdot w}{t}\right)\right) + 0.255 \cdot \left(\frac{t}{w}\right)^2\right]$

$$ZSTR_skny(b, w, t, er) := \frac{60}{\sqrt{er}} \cdot \ln\left(\frac{4 \cdot b}{\pi \cdot ZSTR_K1(w, t)}\right)$$

For wide traces
$$(w/b > 0.35)$$

ZSTR_K2(b,t) := $\left[\frac{2}{1-\frac{t}{b}} \cdot \ln\left(\frac{1}{1-\frac{t}{b}}+1\right) - \left(\frac{1}{1-\frac{t}{b}}-1\right) \cdot \ln\left[\frac{1}{\left(1-\frac{t}{b}\right)^2}-1\right]\right]$

$$ZSTR_wide(b, w, t, er) := \frac{94.15}{\frac{w}{b}} + \frac{ZSTR_K2(b, t)}{\pi} \cdot \frac{1}{\sqrt{er}}$$

Composite formula picks skinny or wide model depending on w/b ratio:

 $ZSTRIP(b, w, t, er) := if(w > .35 \cdot b, ZSTR_wide(b, w, t, er), ZSTR_skny(b, w, t, er))$

Rarely are the two parameters h1 and h2 equal in practice. The more common case is an assymetric stripline having the conducting trace offset to one side.

Offset, or asymmetric, stripline characteristic impedance ($\Omega)$ (no accuracy guaranteed):

 $\mathsf{ZOFFSET}(h1,h2,w,t,er) \coloneqq \frac{2 \cdot \mathsf{ZSTRIP}(2 \cdot h1 + t,w,t,er) \cdot \mathsf{ZSTRIP}(2 \cdot h2 + t,w,t,er)}{\mathsf{ZSTRIP}(2 \cdot h1 + t,w,t,er) + \mathsf{ZSTRIP}(2 \cdot h2 + t,w,t,er)}$

Propagation delay of stripline (s/in.):

$$PSTRIP(er) := 84.72 \cdot 10^{-12} \cdot \sqrt{er}$$

(same formula for centered or offset stripline)

Inductance of stripline (H):

 $LSTRIP(b, w, t, x) := PSTRIP(1.) \cdot ZSTRIP(b, w, t, 1.) \cdot x$

In the equation above, we can assume a relative permittivity of 1.; it doesn't affect the answer.

Inductance of offset stripline (H):

$$LOSTRIP(h1, h2, w, t, x) := PSTRIP(1.) \cdot ZOFFSET(h1, h2, w, t, 1.) \cdot x$$

Capacitance of stripline (F):

 $CSTRIP(b, w, t, er, x) \coloneqq \frac{PSTRIP(er)}{ZSTRIP(b, w, t, er)} \cdot x$

In the equations above and below, we must use the relative permittivity.

Capacitance of offset stripline (F):

 $COSTRIP(h1,h2,w,t,er,x) \coloneqq \frac{PSTRIP(er)}{ZOFFSET(h1,h2,w,t,er)} \cdot x$

SLINE2.MCD -- 9/7/2001 -- p.3

Example stripline calculations

Ground plane separation (in.)	B := .020	
Width of trace (in.)	W := .006	
Thickness of trace (in.)	T := .00137	(1-oz copper plating weight)
Length of wire (in.)	X := 11.000	1 3 3 4

Relative electric permeability (affects capacitance, but not inductance) er := 4.5

Impedance $\left(\Omega\right)$:

ZSTRIP(B, W, T, er) = 51.4371

Total inductance (H):

LSTRIP(B, W, T, X) = 1.0169×10^{-7}

Same result in nH:

LSTRIP(B, W, T, X) $\cdot 10^9 = 101.686$

Inductance per in. (H):

LSTRIP(B, W, T, 1) = 9.2442×10^{-9}

Total capacitance (F):

CSTRIP(B, W, T, er, X) = 3.8433×10^{-11}

Same result in pF:

$$CSTRIP(B, W, T, er, X) \cdot 10^{12} = 38.4334$$

Capacitance per in. (F):

$$CSTRIP(B, W, T, er, 1) = 3.4939 \times 10^{-12}$$

Tolerance effects

$$ZOFF_TOL(h1, dh1, h2, dh2, w, dw, t, er, der) := \begin{pmatrix} ZOFFSET(h1 + dh1, h2 + dh2, w - dw, t, er - der) \\ ZOFFSET(h1, h2, w, t, er) \\ ZOFFSET(h1 - dh1, h2 - dh2, w + dw, t, er + der) \end{pmatrix}$$

$$\alpha := \text{ZOFF}_\text{TOL}(.007, .002, .032, .002, .008, .002, .0015, 4.5, .1)$$

$$\alpha = \begin{pmatrix} 64.0566\\51.7263\\39.228 \end{pmatrix}$$
REFL(x,z) :=
$$\begin{pmatrix} \frac{z - x_0}{z + x_0}\\ \frac{z - x_1}{z + x_1}\\ \frac{z - x_2}{z + x_2} \end{pmatrix}$$
REFL(α , 50) =
$$\begin{pmatrix} -0.1232\\-0.017\\0.1207 \end{pmatrix}$$

SLINE2.MCD -- 9/7/2001 -- p.5